

INTRODUCTORY REMARKS ON TWO PHASE FLOW

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The study of simultaneous flow of two phases of matter is a subject of recent interest due to its importance in various industrial applications, nuclear problems, automobiles, aerospace science and combustion processes. This study may be divided into two groups: the first group consists of the flow problem of the mixture of two phases of four states of matter viz; solid (pseudo fluid), liquid, gas and plasma (ionised gas) mixed homogeneously or inhomogeneously. The second group consists of the flow problems in which the interaction between two phases of matter through their interface is considered. We may classify the flow problems of the two phase flow into the following classes:

- (i) flow of the mixture of liquid and gas.
- (ii) flow of the mixture of solid particles and liquid.
- (iii) flow of the mixture of gas and solid particles.
- (iv) flow of the mixture of liquid and plasma.
- (v) flow of the mixture of solid particles and plasma.
- (vi) flow of the mixture of ordinary gas and ionised gas etc..

Before considering equations of two phase flow we go through the general equations governing flow of a gas.

1- General Equations Governing the Flow of a Gas:

In the study of fluid flow, the fundamental equations governing the

motion of the fluid are (Courant and Friedrichs (1948), Pai (1959), Zeldovich and Raizer (1967)).

(i) Equation of state which connects the temperature T , the pressure p and the density ρ of the fluid. For a perfect gas the equation may be written as

$$(1.1) \quad p = \rho R' T$$

R' being the gas constant

(ii) Equation of continuity which expresses the conservation of mass of the fluid. If q represents the velocity of the fluid at any time t , the equation of continuity may be written as,

$$(1.2) \quad \frac{\partial \rho}{\partial t} + \text{div}(\rho q) = 0$$

(iii) Equation of motion which expresses the relation of conservation of momentum in the fluid. Neglecting the body forces and considering only inertial and pressure forces, the equation of motion may be written as

$$(1.3) \quad \frac{Dq}{Dt} = - \text{grad } p$$

where $\frac{D}{Dt}$ is the usual mobile operator given by

$$(1.4) \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + (q \cdot \nabla)$$

(iv) Equation of energy which expresses the conservation of energy in the fluid. This can be expressed as

$$(1.5) \quad \frac{De}{Dt} + p \frac{D}{Dt} \left(\frac{1}{\rho} \right) = Q$$

where e and Q are internal energy and the energy generated by external sources per unit time per unit mass of the fluid. In the particular case of thermodynamic equilibrium and in the absence of external heat sources, this equations is equivalent to the entropy equation,

$$(1.6) \quad \frac{DS}{Dt} = 0$$

which is the same as the adiabatic flow condition. S is defined as entropy per unit mass of the fluid.

The speed of sound enters the gasdynamic equation as the velocity of propagation of small disturbances. In the limiting case, where changes $\Delta\rho$ in density and Δp in the pressure, accompanying the fluid motion are very small in comparison to the average values of the density ρ_0 and pressure p_0 and where the flow velocities are small in comparison with the speed of sound c , the gasdynamic equations become acoustic equations describing the propagation of sound waves. Neglecting second order quantities and considering only the plane case of a uniform fluid, we may write the continuity equation and the equation of motion in the following forms,

$$(1.7) \quad \frac{\partial}{\partial t}(\Delta\rho) = -\rho_0 \frac{\partial u}{\partial x}$$

and

$$(1.8) \quad \rho_0 \frac{\partial u}{\partial t} = -\left(\frac{\partial p}{\partial \rho}\right)_s \frac{\partial}{\partial x}(\Delta\rho)$$

where

$$(1.9) \quad \Delta p = \left(\frac{\partial p}{\partial \rho}\right)_s \Delta\rho$$

and u is the velocity of the fluid in the X direction.

Here we have used the fact that the particle motion in the sound wave is isentropic. This isentropic derivative represents the square of the sound speed i.e.

$$(1.10) \quad c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$$

Using the well known thermodynamic formula, the square of the velocity of sound in a perfect gas is given by

$$(1.11) \quad c^2 = \frac{\gamma p}{\rho}$$

where γ , as usual is the ratio of specific heats.

2- Shock Wave and its Formation:

Shock wave is a most conspicuous phenomena which occur in nature. In a supersonic flow of a gas it behaves like a surface of discontinuity across which flow variables i.e. velocity, density, pressure and temperature change abruptly. The occurrence of shock waves is commonly associated with supersonic flight, explosion in air and in astrophysical situations, electric discharges etc. Shock wave has great importance for several practical aeronautical problems. In fact, shock waves may cause sudden change in the aerodynamic behaviour of high speed aircrafts affecting not only their

balance and their stability but also controlling undesirable vibrations.

A simple explanation for the shock formation in an ordinary gas can be given as follows:

Let a piston be moving uniformly into an open ended tube filled with virgin gas. Suppose the continuous motion of the piston is approximated by a set of forward moving pulses each of short duration. When the piston moves the first short movement forward, a small disturbance is propagated forward into the gas at the speed of sound. This small amplitude wave heats the gas slightly, the second pulse will be propagate as another sound wave at a speed slightly in excess of the first one. Similarly the third pulse be propagated at a speed slightly in excess of second and so on. Thus the discrete pulses cause a train of sound waves of even increasing velocity to be propagated through the gas. The tendency is for faster moving rearmost waves to catch up with the slower moving foremost one. In so doing the sound waves coalesce to form a more powerful shock front moving at a speed which is in excess of the local speed of sound.

Actually shock wave is not a discontinuity surface in strict sense but it is a layer of small finite thickness across which the physical properties change continuously. If thickness is small in comparison to some appropriate macroscopic dimension of the flow field the physical relationship may be obtained by an analysis which treats it as discontinuity surface. In this case the assumption of small thickness of layer is a fundamental one. The study

of the physical properties of the fluid within the small but finite thickness of the discontinuity is termed as structure of the shock waves. From a mathematical point of view, a discontinuity can be regarded as the limiting case of very large but finite gradients in the flow variables across a layer whose thickness tends to zero. Since in the dynamics of an inviscid and non-conducting gas there are no characteristic lengths, the existence of arbitrary thin transition layers is possible. In the limit of vanishing thickness these layers reduce to discontinuities. Such discontinuities represent shock waves.

Since discontinuity is infinitesimally thin, no accumulation of mass, momentum or energy can take place within it. Consequently, the fluxes of these quantities on both sides of the discontinuity are equal and therefore, treating the discontinuity in a coordinate system in which it is stationary, we have the following equations

$$(2.1) \quad \rho_1 u_1 = \rho_0 u_0$$

$$(2.2) \quad p_1 + \rho_1 u_1^2 = p_0 + \rho_0 u_0^2$$

$$(2.3) \quad e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = e_0 + \frac{p_0}{\rho_0} + \frac{u_0^2}{2}$$

Where u , ρ , p and e are the particle velocity, density, pressure and internal energy per unit mass and subscripts 0 and 1 indicate values of the quantities just ahead and behind the discontinuity surface. These equations (2.1) - (2.3) are known as shock conditions across a

normal shock in an ideal gas. The flow ahead of the shock must be supersonic while behind the shock it may be supersonic or subsonic depending on the inclination of the incident stream to the normal of the wave. If the incident stream is parallel to the normal of the wave, the flow behind the shock is always subsonic.

Hugoniot (1889) first showed that in absence of viscosity and heat conduction conservation of energy implies conservation of entropy in continuous flow and also implies change of entropy across a shock. Rayleigh (1910) pointed out that the entropy must increase in crossing a shock front and that for this reason, a rarefaction shock wave can not occur in a perfect gas. In most cases of the fluid flow we consider negligible viscosity and heat conduction, but when large gradients of temperature and velocity develop, the effects of viscosity and heat conduction become important. Viscosity and thermal conductivity present themselves as diffusion phenomenon arising respectively from the molecular transfer of momentum and energy and they have the effect of smoothing out the discontinuity. By now, the theory of shock waves in homogeneous and non-homogeneous conducting or non conduction media has been extensively studied by several authors such as Courant and Friedrichs (1948), Taylor (1950), Whitham (1958), Sedov (1959), Pai (1959), Stanykovich (1960), Kompaneets (1960), Sakurai (1965), Zeldovich and Raizer (1967), Hayes (1968), Laumbach and Probstein (1969), Korobeinikov (1971), Whitham (1974), Mishkin and Fujimoto (1978), Ojha and

Onkar (1993), Ojha and Srivastava (2011) and so on.

3. Basic Equations in a Mixture of Gas and Small Solid Particles:

When a large number of solid particles flow in a fluid and the velocity of the fluid is sufficiently large, the behaviour of such solid particles is similar to the fluid. Assuming these solid particles as pseudo fluid we can consider the flow of the fluid as a mixture of real fluid (gas or liquid) and the pseudo fluid of solid particles. To discuss the flow of the above mixture, consider an element of the mixture of mass M occupied by volume V in the flow such that (Marble (1970), Murray (1965), Soo (1967), Pai (1977), Rudinger (1980) etc)

$$(3.1) \quad M = M_g + M_p$$

$$(3.2) \quad V = V_g + V_p$$

where subscripts g and p refer to the values for the gas and the solid particles respectively.

Suppose each particle is a sphere of radius r_p and n_p is the number of solid particles per unit volume at a point in the flow field, then volume V_p occupied by all solid particles in the volume V is given by

$$(3.3) \quad V_p = n_p V \tau_p$$

where $\tau_p = \frac{4}{3} \pi r_p^3$ i.e. volume of each solid particle. The mass of the solid particles in the volume V of the mixture is

$$(3.4) \quad M_p = m_p n_p V$$

where m_p is the mass of each particle.

The species density of the solid particles is defined as

$$(3.5) \quad \rho_{sp} = \frac{M_p}{V_p} = \frac{m_p}{\tau_p}$$

Hence the species density of solid particles is a constant for a given flow.

The partial density of the solid particles in volume V is defined as

$$(3.6) \quad \rho_p = \frac{M_p}{V} = m_p n_p = Z \rho_{sp}$$

where Z represents the fraction of volume of solid particles in the mixture. This Z is one of the important variables in the treatment of two phase flow of a gas and small solid particles and is defined as

$$(3.7) \quad Z = \frac{V_p}{V} = n_p \tau_p$$

Thus, ρ_p the partial density of the solid particles is also one of the fundamental variables in analysis of the mixture flow problems and is proportional to Z or n_p .

Similarly, we have also the species density of the fluid and the partial density of the fluid too. The species density of the real fluid is defined as

$$(3.8) \quad \rho_g = \frac{M_g}{V_g}$$

and the partial density of the real fluid is defined as

$$(3.9) \quad \bar{\rho}_g = \frac{M_g}{V} = \frac{M_g}{V_g} \frac{V_g}{V} = \frac{M_g}{V_g} \left(\frac{V - V_p}{V} \right) = (1 - Z) \frac{M_g}{V_g} = (1 - Z) \rho_g$$

Suppose in the mixture of a gas and small solid particles, gas is an ideal gas and therefore from the equation of state for a perfect gas

$$(3.10) \quad p_g = R' \bar{\rho}_g T_g$$

where p_g and T_g are the partial pressure and partial temperature of the gas in the mixture and R' is the gas constant.

Let the total pressure of the mixture is the sum of the partial pressure of the gas p_g and partial of the pseudo-fluid of solid particles p_p , then

$$(3.11) \quad p = p_g + p_p$$

From (3.9) and (3.10), we may write

$$(3.12) \quad p_g = R'(1 - Z) \rho_g T_g$$

The total pressure p may be also defined as

$$(3.13) \quad p = R' \rho_g T_g$$

since ρ_g is species density.

From (3.11), (3.12) and (3.13) we define that

$$(3.14) \quad p_g = (1 - Z)p \text{ and } p_p = Zp$$

The equation of state for the pseudo-fluid of solid particles is defined simply as

$$(3.15) \quad \rho_{sp} = \text{constant.}$$

In case of thermodynamic equilibrium, $T_p = T_g = T$. The density of the mixture as whole is given by

$$(3.16) \quad \rho = Z\rho_{sp} + (1-Z)\rho_g = \rho_p + \bar{\rho}_g$$

The mass concentration of the pseudo fluid particles is defined as

$$(3.17) \quad k_p = \frac{\rho_p}{\rho} = \frac{Z\rho_{sp}}{\rho}$$

In equilibrium flow, k_p is constant in the whole flow field and therefore from equation (3.17)

$$(3.18) \quad \frac{Z}{\rho} = \text{constant}$$

in the whole flow field.

Also, from equations (3.16) and (3.17), we can write

$$(3.19) \quad Z = \frac{k_p}{(1-k_p)G + k_p}$$

where

$$(3.20) \quad G = \frac{\rho_{sp}}{\rho_g}, \text{ called compressibility}$$

of the mixture.

Again, in case of thermodynamic equilibrium, writing $T = T_g = T_p$ and using the equations (3.13), (3.16) and (3.17) we can define a relation between the pressure and density of the mixture as whole

$$(3.21) \quad p = \frac{(1-k_p)}{(1-Z)} \rho R' T$$

This equation is known as equation of state for a mixture of gas and small solid

particles as whole. We can write (3.21) in the form

$$(3.22) \quad p = \frac{R_m \rho T}{(1-Z)}, \quad R_m = (1-k_p)R'$$

where R_m is effective gas constant of the mixture. It is interesting to notice that if the volume fraction Z of the solid particles is negligibly small, the perfect gas law holds with effective gas constant R_m . For many engineering problems, we do have a very small value of Z but k_p is not negligible in comparison with unity. On the other hand if Z is not negligibly small in comparison with unity, the volume fraction Z of the solid particles does affect the equation of state of the mixture as a whole because Z is a function of ρ .

The internal energy e of the mixture per unit mass is related to the internal energies of the two species by the following relation

$$(3.23) \quad \rho e = Z\rho_{sp}c_{sp}T_p + (1-Z)\rho_g c_v T$$

or

$$(3.24) \quad e = k_p c_{sp} T_p + (1-k_p) c_v T$$

where c_v is the specific heat of the gas at constant volume and $c_{sp} = c_s + c_{vp}$. Here c_{vp} being an effective specific heat at constant volume of the pseudo-fluid of solid particles due to random translational motion and c_s is the specific heat of the solid particles due to the internal degree of freedom. For thermodynamic equilibrium condition,

we have the specific heat of the mixture at constant volume is given by c_{vm} , where

$$(3.25) \quad c_{vm} = k_p c_{sp} + (1 - k_p) c_v, \quad c_{sp} \text{ and } c_v \text{ are taken as constant.}$$

The specific heat of the mixture at constant pressure is given by

$$(3.26) \quad c_{pm} = k_p c_{sp} + (1 - k_p) c_p$$

when c_p is the specific heat of the gas at constant pressure.

The specific heats of the mixture are independent of the volume fraction Z but depend on the mass concentration k_p of solid particles. The ratio of specific heats of the mixture is

$$(3.27)$$

$$\Gamma = \frac{c_{pm}}{c_{vm}} = \frac{(1 - k_p)c_p + k_p c_{sp}}{(1 - k_p)c_v + k_p c_{sp}} = \frac{\gamma \left(1 + \delta \frac{\beta'}{\gamma}\right)}{1 + \beta' \delta} = \frac{\gamma + \delta \beta'}{1 + \delta \beta'}$$

where

$$(3.28) \quad \gamma = \frac{c_p}{c_v}, \beta' = \frac{c_{sp}}{c_v}, \delta = \frac{k_p}{1 - k_p}$$

The ratio Γ is always smaller than the ratio γ of the gas if k_p is different from zero. As $k_p = 0, \Gamma = \gamma$.

From (3.25) and (3.26), we obtain

$$(3.29) \quad c_{pm} - c_{vm} = (1 - k_p) R'$$

where $c_p - c_v = R'$

Again from (3.27) and (3.29), we have

$$(3.30) \quad c_{vm} = \frac{(1 - k_p) R'}{\Gamma - 1},$$

$$c_{pm} = \frac{(1 - k_p) R' \Gamma}{\Gamma - 1}$$

and therefore

$$(3.31) \quad e = c_{vm} T = \frac{(1 - k_p) R' T}{\Gamma - 1} = \frac{(1 - Z) p}{(\Gamma - 1) \rho}$$

The first law of thermodynamics for the mixture is given by

$$(3.32) \quad dQ = de - \frac{p}{\rho^2} d\rho$$

where dQ is the amount of heat added per unit mass of the mixture

In case of isentropic flow, $dQ=0$ and therefore

$$(3.33) \quad de = \frac{p}{\rho^2} d\rho$$

This equation is the energy equation of the mixture as a whole. using equations (3.21), (3.31) and (3.33) we obtain

$$(3.34) \quad \frac{1}{(\Gamma - 1) T} \frac{dT}{T} = \frac{1}{(1 - Z) \rho} \frac{d\rho}{\rho}$$

since $Z = \frac{k_p \rho'}{\rho_{sp}}$ for constant k_p

integration of (3.34) gives

$$(3.35) \quad T \left(\frac{\rho}{1 - Z} \right)^{-(\Gamma - 1)} = \text{constant}$$

If $Z \ll 1$, the isentropic change of state of the mixture has similar relation as that for a pure gas with an effective ratio of the specific heats Γ . In general the volume fraction Z has some influence on the isentropic change of the mixture.

Similarly from equation (3.21) for a given k_p we have

$$(3.36) \quad \frac{dp}{p} = \frac{dT}{T} + \frac{1}{1-Z} \frac{d\rho}{\rho}$$

putting the value of $\frac{dT}{T}$ from (3.34) and

then integrating,

we obtain

$$(3.37) \quad p \left(\frac{\rho}{1-Z} \right)^{-\Gamma} = \text{constant.}$$

Again if $Z \ll 1$, equation (3.37) is identical in form for the corresponding relation of pure gas with an effective ratio of specific heats.

The speed of sound in equilibrium state of the mixture can be obtained from (3.37) as a_M where

$$(3.38) \quad a_M^2 = \frac{dp}{d\rho} = \frac{\Gamma p}{\rho(1-Z)} = \frac{\Gamma R_m T}{(1-Z)^2}$$

The ratio of the equilibrium sound speed of the mixture a_M to that of the gas c is

$$(3.39) \quad \frac{a_M}{c} = \left(\frac{\Gamma}{\gamma(1-Z)} \right)^{\frac{1}{2}}$$

where $c = \left(\frac{\gamma p}{\rho} \right)^{\frac{1}{2}} = (\gamma R T)^{\frac{1}{2}}$

4. One Dimensional Fundamental Equations of Motion and Shock Conditions in a Mixture of Gas and Solid Particles:

The Equation of Continuity:

The unsteady one dimensional equation of continuity for a mixture of gas and

small solid particles is given by (Pai (1977), Conforto (2001), Steiner and Hirschler (2002), Ojha and Srivastava (2007) etc)

$$(4.1) \quad \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{i \rho u}{r} = 0$$

where $i = 0, 1$ and 2 corresponding to the plane, cylindrical and spherical symmetry respectively; ρ is the density of the mixture as whole, u the flow velocity of the mixture and r and t are space and time coordinates respectively.

The Equation of Motion:

In absence of viscosity and heat conduction the equation of motion for the unsteady one-dimensional flow of the mixture of gas and small solid particles can be written as

$$(4.2) \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0$$

where p is the pressure of the mixture

The Equation of Energy:

The energy equation for unsteady one-dimensional flow of the mixture of a gas and small solid particles in which the viscous stress and heat conduction are assumed to be negligible can be written as

$$(4.3) \quad \frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0$$

where e is the internal energy per unit mass of the mixture of gas and small solid particles.

If heat conduction and radiation heat flux are taken into account then equation of energy can be written as

(Gretler and Regenfelder (2002), Ojha and Srivastava (2011)).

$$(4.4) \quad \frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r^i} \frac{\partial}{\partial r} (r^i F) = 0$$

where F is the total heat flux and is given by

$$(4.5) \quad F = F_C + F_R$$

where F_C is conduction heat flux and F_R , the radiation heat flux. $i=0,1,2$ for planar, cylindrical and spherical symmetry respectively.

The conduction heat flux F_C is given by Fourier's law of heat conduction which is

$$(4.6) \quad F_C = -K \frac{\partial T}{\partial r}$$

where K is the coefficient of thermal conductivity of the gas. The radiation heat flux F_R is given by (Pomraning (1973), Gretler and Regenfelder (2002))

$$(4.7) \quad F_R = -\frac{4}{3} \left(\frac{\sigma}{\alpha_R} \right) \frac{\partial T^4}{\partial r}$$

where σ is the Stefan-Boltzmann constant and α_R is the Roseland mean absorption coefficient.

The thermal conductivity K and the absorption coefficient α_R of the medium are assumed to vary with temperature and density (Ghoniem et al (1982), Gretler and Regenfelder (2002)) as

$$(4.8) \quad K = K_0 \left(\frac{T}{T_0} \right)^{\beta_C} \left(\frac{\rho}{\rho_0} \right)^{\delta_C};$$

$$\alpha_R = \alpha_{R_0} \left(\frac{T}{T_0} \right)^{\beta_R} \left(\frac{\rho}{\rho_0} \right)^{\delta_R}$$

where subscript 'o' denotes a reference state.

The generalised shock conditions in a mixture of gas and small solid particles are

$$(4.9) \quad \rho_2 (U - u_2) = \rho_1 U$$

$$(4.10) \quad p_2 + \rho_2 (U - u_2)^2 = p_1 + \rho_1 U^2$$

$$(4.11)$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} (U - u_2)^2 = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} U^2$$

$$(4.12) \quad \frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1}$$

Here, it is assumed that the viscous stress and heat conduction of the mixture are negligible.

If conduction and radiation heat flux are taken into account then the shock condition (4.11) takes the form

$$(4.13)$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2} (U - u_2)^2 = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2} U^2 + \frac{F_2}{\rho_1 U}$$

where the subscripts 2 and 1 denote conditions immediately behind and ahead of the shock respectively and U denotes the shock velocity.

Recently shock waves in a mixture of small solid particles and a gas have been studied extensively by several authors, e.g. Suzuki et al (1975) Pai et al (1980), Miura and Glass (1985), Jena and Sharma (1999), Vishwakarma (2000),

Gretler and Regenfeldor (2001, 2002), Steiner and Hirschler (2002), Vishwakarma and Pandey (2003), Ojha and Srivastava (2007) etc.

5. Thermodynamics of the Non-Ideal Gas and a Mixture of a Gas and Small Solid Particles:

At high temperature the assumption of gas to be perfect is in general not true. The perfect gas law can be applied only to actual gases with sufficient accuracy. This approximation may however be inadequate in a situation such as that arises in the case of strong explosion. It is then necessary to take into account of the deviations of an actual gas from the ideal state which results from interaction between its component molecules.

In general the thermal equation of state for an actual gas is given by (Cambel, Duclos and Anderson (1963))

$$(5.1) \quad p = \theta \rho R'T$$

where θ is the departure coefficient depending on the existing physical conditions. For a perfect gas $\theta = 1$

From statistical mechanics of the gas the thermal equation of state may be written in the form (Landau and Lifshitz (1958))

$$(5.2) \quad p = \rho R'T (1 + b_1 \rho + c_1 \rho^2 + \dots)$$

called virial equation of state. The temperature dependent quantities b_1, c_1 etc. are called the second, third etc. virial coefficients. At high temperature the coefficients b_1 and c_1 tend to constant values equal to \bar{b} and $\frac{5}{8}\bar{b}$ respectively.

In many physical cases the introduction of the second coefficient is sufficient. For

gases $\bar{b} \rho \ll 1$, \bar{b} being the internal volume of the gas molecules, it is sufficient to consider the equation of state in the form (Anisimov and Spiner (1972), Ranga Rao and Purohit (1976), Ojha and Tiwari (1993), Roberts and Wu (2003), Ojha and Srivastava (2011))

$$(5.3) \quad p = \rho R'T (1 + \bar{b} \rho)$$

or equivalently

$$(5.4) \quad p = \frac{R'T}{V - \bar{b}} \quad , \quad V = \frac{1}{\rho} \quad \text{being}$$

specific volume of the gas.

If e is the internal energy per unit mass of the gas, then from thermodynamics,

$$(5.5) \quad \left(\frac{\partial e}{\partial V} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_V - p$$

Using the equation of state (5.3) in (5.5) we get $\left(\frac{\partial e}{\partial V} \right)_T$ is negligible which shows

that

$$(5.6) \quad e = c_v T \quad ; \quad c_v \text{ is the specific heat at constant volume.}$$

Using equation (5.5) in the first law of thermodynamics, we have

$$(5.7) \quad c_p - c_v = T \left(\frac{\partial p}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_p$$

where c_p is the specific heat of the gas at constant pressure.

Using (5.3) in equation (5.7) we get

$$(5.8) \quad c_p - c_v = \frac{R'(1 + \bar{b}\rho)^2}{(1 + 2\bar{b}\rho)} \approx R'$$

neglecting the higher powers of $\bar{b}\rho$

Equation (5.8) implies that

$$(5.9) \quad c_v = \frac{R'}{\gamma - 1}$$

The equation (5.3), (5.4) and (5.9) give the internal energy e as a function of p and ρ i.e.

$$(5.10) \quad e = \frac{p}{\rho(1 + \bar{b}\rho)(\gamma - 1)} \approx \frac{p(1 + \bar{b}\rho)}{\rho(\gamma - 1)} \approx \frac{p(V - \bar{b})}{\gamma - 1}$$

The speed of sound c_n may be calculated from (5.3) as follows:

$$(5.11) \quad c_n^2 = \frac{dp}{d\rho} = \frac{(1 + 2\bar{b}\rho)\gamma p}{(1 + \bar{b}\rho)p}$$

For a gas obeying virial equation $p = \frac{R'T}{V - \bar{b}}$ which undergoes an isentropic process

$$(5.12) \quad p(V - \bar{b})^{\gamma^*} = \text{Const.}$$

where $\gamma^* = \left(1 + \frac{R'}{c_v}\right)$ with constant c_v .

The equation of state for a mixture of a non-ideal gas and pseudo-fluid of small solid particles is given by (Vishwakarma and Nath (2009))

$$(5.13) \quad p = \frac{1 - k_p}{1 - Z} [1 + b\rho(1 - k_p)] \rho R'T$$

The internal energy per unit mass of the mixture of a non-ideal gas and small solid particles is given by

$$(5.14) \quad e = \frac{p(1 - Z)}{\rho(\Gamma - 1)[1 + b\rho(1 - k_p)]}$$

If we consider the mixture as a homogeneous medium, the first law of thermodynamics for the mixture gives

$$(5.15) \quad dQ = de - \frac{1}{\rho^2} p d\rho$$

where dQ is the heat addition to the mixture per unit mass.

For isentropic change of state of the mixture we have $dQ=0$, therefore we can derive an equation

$$(5.16) \quad \frac{1}{\Gamma - 1} \frac{dT}{T} = \frac{[1 + b\rho(1 - k_p)] d\rho}{1 - Z} \frac{1}{\rho}$$

From equation (5.13), we have

$$(5.17) \quad \frac{dp}{p} = \left[\frac{\Gamma}{(1 - Z)\rho} + \frac{(\Gamma - 1)b(1 - k_p)}{1 - Z} + \frac{b(1 - k_p)}{1 + b\rho(1 - k_p)} \right] d\rho$$

or

$$(5.18) \quad p \left(\frac{\rho}{1 - Z} \right)^{-\Gamma} \frac{1}{[1 + b\rho(1 - k_p)](1 - Z)^{-(\Gamma - 1)b(1 - k_p)}} =$$

constant

The equilibrium speed of sound of the mixture of a non-ideal gas and small solid particles can be obtained as

$$(5.19) \quad c_n^* = \left(\frac{dp}{d\rho} \right)_s^{\frac{1}{2}} = \left[\frac{\{\Gamma + (2\Gamma - Z)b\rho(1 - k_p)\} p}{(1 - Z)\{1 + b\rho(1 - k_p)\} \rho} \right]^{\frac{1}{2}}$$

neglecting $b^2 \rho^2$

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